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Using inventory to handle risks in the supply of oil to Nepal

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Abstract

Nepal's unique geographical features, frequent political disturbances, strikes, a limited and complicated road network, frequent road breakdowns, government interventions as well as a volatile international oil market make the supply chain of Nepal Oil Corporation (NOC) rather unique. We analyze different risks in the NOC supply chain and discuss what can be done to find inventory strategies that handle these risks without a particular focused on one specific scenario.

Keywords: Supply Chain, Stochastic programming, Uncertainty, Linear programming, Inventory

1 Introduction

Nepal Oil Corporation (NOC), established by the Nepal government in 1970, is a public enterprise to import, store and distribute petroleum products in the country. NOC maintains monopoly in this market. Nepal being a landlocked country, the entire oil procurement is done through India.

Earlier, the supply of oil to Nepal was handled, in a rather random fashion, by a few private companies. To regulate these unplanned purchases as well as the resulting distribution, the Nepalese government established NOC. This monopoly state of NOC was not created to make profit but to have effective distribution at reasonable prices for the customers. We can see this from the losses it has suffered when oil prices increased in the last decade. NOC has over the last five years accumulated a loss of 12 billion rupees (approx 300 million US Dollars). The government has decided to extend loans of 1.7 billion rupees on its guarantee to NOC to pay dues to the Indian supplier. As of June 2007, `http://ekantipur.com` estimated that NOC owes 4.5 billion rupees to the Indian Oil Corporation (IOC) . Long queues in front of petrol pumps have become a common phenomenon in Kathmandu and in the rest of the country, which is of more concern to the government and to the general customers, than NOC's losses.

Oil being a sensitive product, NOC finds it difficult to operate efficiently due to government interventions. Pricing and procurement are political and bilateral issues, which are dictated by the government and the bilateral agreement between Nepal and India. The major job in NOC is that of operational activities which includes storing and distributing oil products. NOC sees possibilities of improving its performance by managing the operational activities of its supply chain in a better way.

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The country's unique geographical features, frequent political disturbances, strikes, limited and complicated road network, frequent road breakdowns, government interventions and a volatile international oil market make this supply chain a very complicated one. A study of NOC's distribution network motivated us to better understand the risks and uncertainties associated with the supply chain and how these can be managed. We shall focus on how long-term inventories can be used to manage the risks. This way of management, being rather standard in general, is particularly involved for a company consistently short on foreign currencies.

The paper is organized as follows. The next section explains the supply chain of NOC and the major players in this network; the third section deals with risks in a supply chain. The fourth section dwells on a linear programming formulation of NOC's distribution problem and its possible extensions. The fifth deals with a stochastic version of the model and the scenarios are discussed. The sixth section explains the result of the optimization problem while, finally, the seventh section is the conclusion of the paper.

2 NOC's supply chain and its distribution network

The supply chain for oil distribution in Nepal is unique in the sense that there are not many players and levels in this network. The focal company in this analysis does not have full control of the complete supply chain, as it is heavily dependent on the agreement between India and Nepal at government level and many other external parties. NOC, our focus company, can only increase its efficiency by taking what is given and optimize the network from there. The paper focuses on the distribution network between the refineries and depots of the IOC and the depots of NOC.

2.1 The players in NOC's supply chain

The supply chain starts with the International market where crude oil is bought. The oil market is among the most volatile commodity markets and is extremely sensitive to international events. Over the last decade, the international prices have increased from about 18 dollars to around 140 dollars per barrel, and then dropped to the present level of about 50.

The *Indian Oil Corporation* buys crude oil and processes it into products. NOC imports oil from refineries/depots of IOC situated at different places in India. IOC determines prices for the oil products that include costs of crude oil at Kolkata port, the transportation cost to the refineries and the refining charges. In particular the refining charges are negotiable. *NOC* has 12 depots in Nepal with total storage capacity of around 70300 kilolitre. Presently NOC fills only 15 to 18 percent of the full capacity due to its financial constraints. This covers just a few days of consumption. Nearly 1200 trucks, owned by *independent transport companies*, are on contract to distribute the oil products. Most of the trucks are under trucks associations, which are mostly at regional level, and they do not like to operate outside their own regions. There are approximately 1900 independently owned distribution points throughout the nation. These points cater to the customers' demand of all oil products except aviation fuel. Aviation fuel is distributed directly by NOC. This network of *distributors* makes it possible to reach the end *customers* of NOC. A population of around 27 million, all dependent on oil in some form, directly or indirectly, puts a lot of pressure on this supply chain. Present total demand of oil products per year is around 0.8 million kilolitre. Sixty-five percent of the total demand originates from the Kathmandu valley. Overall there is an annual growth rate of around 10 percent in the demand for petroleum products. The network structure of the supply chain is illustrated in Figure 1.

In Figure 1, refineries are owned by IOC and are situated at different places in India. Presently

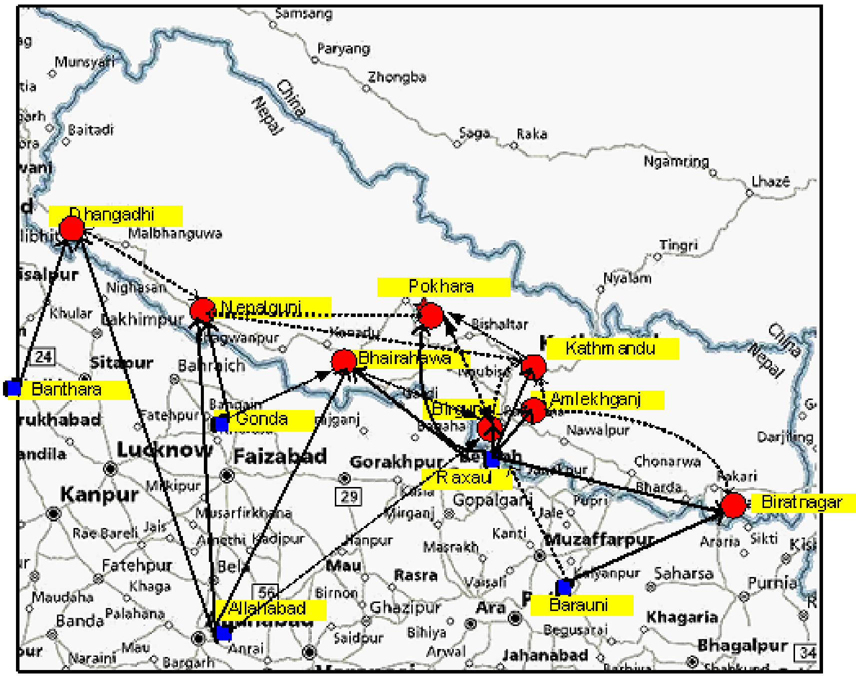

Figure 1: Network structure. The circles denote the NOC depots in Nepal, while the squares denote the IOC refineries. Note that the only low-lands in Nepal are along the Indian border, the rest of the country consists of mountains and the highly elevated Kathmandu Valley.

NOC uses five refineries or depots of IOC to buy petroleum product. The transportation of petroleum product from the refineries to different depots of NOC is done using trucks. The depots in Nepal are served directly or via some of the other depots. The eight most important depots will be in the focus of this study. Some of the depots here act as transshipment points from which smaller sized tankers are used to the mountainous regions. There are several road links between the refineries/depots and depots situated in Nepal, but only few of these links are actually used. For an idea of the actual road links between the depots of NOC, see `http://www.lirung.com/map/map_road/Nepal_Road_Map_e.html`.

3 Risks in the distribution network

Supply chain thinking has become critical in today's business environment. The distribution network, an element of the supply chain, creates links between two tiers of supply chain members. Most of the time, these links are crucial for the success of a business. Due to the dynamic nature of the business environment, there are lots of uncertainties present in the network. Managers need to identify and manage the risks created by these uncertainties that hamper the performance. The network is mostly analyzed from the buyers' point of view with ideas from marketing and network design, in terms of cost and reach. One of the major issues in a network is the risk of failure. The risk to a network is multi-faceted and cannot be captured with a single number, see March and Shapira (1987).

There are three different types of uncertainty in supply chains: demand uncertainty, supply uncertainty and technology uncertainty, see Davis (1993). These uncertainties bring risk to the supply chain. The risk concept has been extensively studied in business contexts and in all studies supply risk is regarded as one of the major risks—see Zsidisin (2003). By studying the supply risk we capture most of the risk arising from the tangible and intangible features of the supply chain.

Meulbrook (2000) defines supply risk as any type of risk that ‘adversely affects inward flow of any type of resource to enable operations to take place’ whereas Zsidisin, Panelli, and Upton (2000) define supply risk as ‘the transpiration of significant and/or disappointing failures with inbound goods and service’. Zsidisin (2003), from case studies with purchasing organizations involved in supply risk management, concludes that supply risk can be defined as the probability of an incident associated with inbound supply from individual suppliers or the supply market occurring, in which the incident results in the inability of the purchasing firm to meet customer demand or causes threats to customer life and safety.

Most research work on optimization of networks has focused on uncertain demand, but very few have analyzed uncertainty from the supply side. This paper analysis risks due to such as random supply and arc failures.

The network facing NOC has both intangible and tangible features. The intangible features are the political relation between the governments of India and Nepal, the international political scenario, the bargaining power of Nepal with India, and Indian foreign policy towards Nepal. The intangible features of a network are difficult to examine and influence, see Harland, Brencheley, and Walker (2003), but still have profound effects on design decisions and performance of the network. We see that changes in the government and/or managers, change the relation between the management and the government which again affects the decision making style. The tangible features are examinable, but they are also to some extent influenced by the intangible features of the network. Thus identifying, defining, and assessing these risks properly for the given network will be a major task.

The breakdown of a source point due to technical fault can be classified as technology uncertainty. In the given case the breakdown in refineries (source point) will result in disruption of supply so this may also be viewed as a source of supply uncertainty. In the present case the breakdown of any part of the roads due to any climatic condition or man-made situation can also be regarded as supply uncertainty. Also political or organizational fallouts may result in blocking or restriction of supply from seller to buyer, which also may be regarded as supply uncertainty.

NOC does not use any linear programming model to plan the supply chain but uses their experience, basic ideas, and simple mathematics to calculate the figures required for ordering, storing and transportation. This model, however, is built in cooperation with NOC, and reflects their way of thinking. Data was also collected from NOC.

4 Modelling philosophy

There are of course many ways to handle the risks in NOC’s supply chain. Reducing those risks that are fully or partly man-made will always have a major focus. Our thinking in this paper is somewhat different. We shall focus on long-term inventory, and study how it changes as the different risks (and other model parameters) change for whatever reason. Inventory as a means of handling risks is particularly involved for a company like NOC which is consistently short on foreign currencies, needed to purchase oil from India.

Obviously, the problem of inventory control while purchasing under a limited period-by-period budget and random disturbances is an infinite horizon problem. The disturbances occur (mostly) independently of each other, and any disturbance can occur at any point in time. As it stands, this problem is not very well suited for stochastic programming. However, we must remember that our main issue is long-term inventory control. That is, we would like to understand how inventory should be controlled when everything runs smoothly in expectation of some disturbance. The expectation is

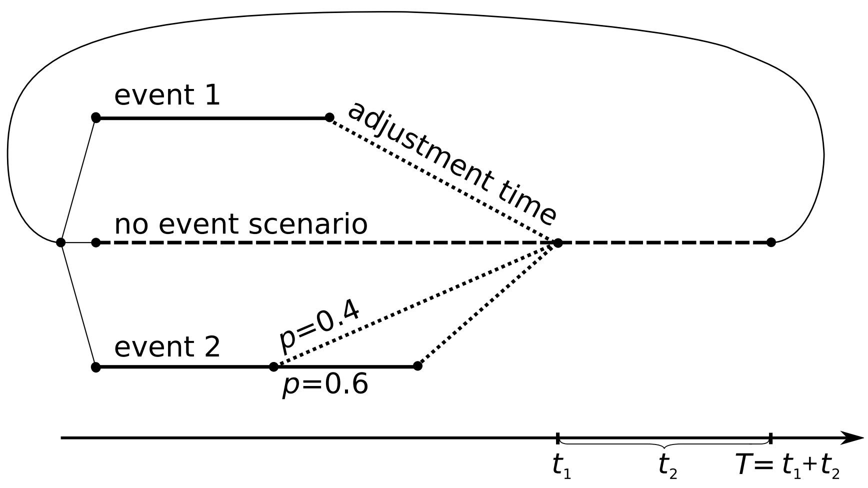

Figure 2: Illustration of events with known and random durations (solid lines), the scenario of no events (dashed line), the period of inventory adjustments (dotted lines), the period of carrying inventory in anticipation of an event (dashed line), and finally the loop.
Event 1 has a known duration, while the duration of Event 2 is stochastic: it has a probability of 40% to finish earlier and 60% of lasting longer then Event 1.

that as the probability of disturbances increases, optimal inventory increases, increasing capital costs. So we expect to see the standard trade-off between inventory costs and shortage costs. But the setting is rather different from a simple inventory model.

We shall first make the somewhat common assumption that only one disturbance can occur at any point in time. Or in other words, that the probability of two disturbances occurring at the same time is so low that we can disregard it. In our case, this assumption is critical.

The present model is not an operational model but rather represents a way to explain the effects of the different random disturbances on long-term inventory. Figure 2 illustrates our modelling. The idea in the model is that we introduce all random events at a particular point in time (arbitrarily denoted 0). At that point inventories and whatever parts of the supply chain that are still working, are used to supply customers. When the events are over, inventory levels must be brought back to their long-term levels (which are our main variables) within time t_1. Thereafter follow t_2 periods of no disturbances. The t_2 periods represent where the model will carry the inventory, anticipating the random events. This is the core of the model – carrying inventory in expectation of some disturbance. The longer t_2, the less likely is an occurrence. As there is also a scenario representing “no disturbance” we have two ways of representing the intensity of disturbances: the choice of t_2 and the probability of the no disturbance scenario. After the model reaches time $T = t_1 + t_2$, we loop the model back to the start. This way of looping back can be useful in stochastic models, see for example Lium, Crainic, and Wallace (2007).

So we shall end up with a three-stage stochastic programming model. First stage variables shall be inventory levels in periods of no disturbance, second and third stage variables are all others. Third stage variables are associated with events with random durations (like Event 2 in Figure 2). We could have let also transportation and purchasing decisions be first stage in the same periods, but have chosen not to: there might be problems of stability of those variables around the end points of the stable periods, and our focus is in any case on the long-term (first-stage) inventories.

It is important to note that the second and third stage variables do not in principle represent operational decisions that can be implemented. The model’s sole focus is on inventories in stable periods.

And even more obviously: The elapse of time in the model does not represent the actual flow of time. We believe this formulation of a three stage model for an infinite horizon problem to be a major contribution of this paper. Of course, the approach works as well for pure two or multiple stages depending on the branching structure of the events.

So we conclude this section by repeating the relationship between our problem understanding and our model. In reality, events occur at random points in time, but with known frequencies (data is available for many of the events). Whenever one occurs, NOC does its best to supply the country with oil using inventory and those parts of the supply chain still functioning. As soon as the event is over, NOC will try to recover the chosen inventory levels. Those inventories will then be carried until the next event starts. So we see that the less likely the events, the more costly the inventories (in the sense that they are more rarely used). In the model, the quiet period of length t_2 (as well as the no-event scenario) represents when NOC waits for the next event. In the model it is known when the events occur. But the model is not allowed to take that into account since we force inventories to be carried at stable levels throughout the quiet period. Once an event occurs, the model will try to supply oil by using inventories and whatever of the supply chain is working. Then, after the event is over, the model forces a rebuild of the inventories, so that they reach the inventories of the stable period by time t_1. Hence, the major variables – the inventories – function the same way in the model as in reality. They are kept in anticipation of events when all is quiet, and used whenever an event occurs. The assumption of no two events taking place at the same time is needed for this modelling to work. In this case this is a reasonable assumption.

5 LP formulation

We formulate the present distribution philosophy of NOC as an LP. This formulation looks into the distribution of oil products from up to five refineries of IOC in India to a number of depots of NOC in Nepal. The objective function minimizes the overall cost of distribution, inventory and penalties for non-delivery. Following present practice, we always source a depot from the nearest refinery. Since not all refineries deliver all products, a given depot might be sourced from several refineries, but there will never be two refineries sourcing the same product to a given depot. Purchasing of products takes place in USD. Since the Rupee is not convertible, NOC cannot transfer any amount into USD. So instead of having a total budget for all activities, or at least minimizing total costs, we have chosen to minimize costs that occur in Rupees under two major constraints: The availability of USD for purchasing and a requirement that the budget is fully used for purchasing.

We start with a basic LP formulation of the problem when everything is normal. It is a kind of deterministic multi-period multi-commodity production and transportation problem with inventory. To address the end-of-horizon problem, we present the model in a circular fashion. This can be done in a simple way by letting the period following period t be $(t+1) \mod (T+1)$ and the previous period $(t+T) \mod (T+1)$, where $0,\ldots,T$ are the time periods in the model.[1] Since we develop the model in a circular fashion we need not provide initial inventories. The model will itself find the appropriate inventory levels, in fact, that is the purpose of the model. This way we avoid ending up analyzing the build-up (transient) stage of the operation, which is quite different from the steady-state, which is our focus.

Since NOC is short on foreign currencies, there will be rejection of demand. We represent that by piece-wise linear penalties. It is crucial not to use simple linear penalties, as that will result in

[1] There seems to be a disagreement about the interpretation of $a \mod b$ if a is negative. Hence, we have chosen to let the period preceding period t be $(t+T) \mod (T+1)$ and not $(t-1) \mod (T+1)$

unreasonable rejections, such as all demand in a given depot on a given day rejected rather than rejections spread over depots and time.

The term "node" might need a brief explanation. In the deterministic version of the problem a node represents all decisions associated with a specific time period (day in our case). Hence, if we start counting both nodes and time at zero, the time associated with node n, denoted $\mathrm{t}(n)$ is always given by $\mathrm{t}(n) = n$. For the stochastic model that will change. We use n and $\mathrm{t}(n)$ also in the deterministic formulation to make the transition to the stochastic model as easy as possible.

Sets

- $\mathscr{N}$ Nodes
- $\mathscr{D}_1$ Depots that can be reached from their nearest refinery in one day.
- $\mathscr{D}_2$ Depots that can be reached from their nearest refinery in two days.
- $\mathscr{D}$ Set of all depots; $\mathscr{D} = \mathscr{D}_1 \cup \mathscr{D}_2$.
- $\mathscr{I}$ Set of intervals for the piece-wise linear penalty for unsatisfied demand.

Input parameters

- H_k Inventory holding cost per unit of product k per unit of time. As this is mainly capital costs, it does not depend on $j \in \mathscr{D}$.
- C_{ik}^n Unit cost of transporting product k to depot i (from its nearest refinery).
- c_{ij}^n Transportation costs between depots i and j inside Nepal.
- P_k Price of product k, measured in USD.
- g_{ik}^n Fairness factor (percentage of demand fulfilled) for each product at each depot.
- d_{ik}^t Demand for product k at depot i at day t.
- M_{ik} Maximum holding capacity of product k at depot i.
- B Budget in USD for buying products over the time horizon $0, \ldots, T$.
- U Total capacity of trucks available each day.
- b_l Lengths of intervals for the piece-wise linear penalty; $l \in \mathscr{I}$
- $N_{ik,l}$ Coefficients for the piece-wise linear penalty for product i at depot k; $l \in \mathscr{I}$.
- $\mathrm{t}(n)$ The time of node n.
- $\mathrm{pa}(n)$ Parent of node n, i.e. the node preceding it. To make the network circular, we set the parent of the first node to be equal to the last node. This will guarantee that $\mathrm{t}\big(\mathrm{pa}(n)\big) = \big(\mathrm{t}(n) + T\big) \mod (T+1)$.

Decision variables (all depend on n, something that will not be repeated throughout)

- x_{jk}^n Amount of product k transported to depot j. We use preprocessing to determine which refinery will be used. Hence there is no "from" index on x.
- w_{ijk}^n Amount of product k sent from depot i to depot j (both within Nepal)
- z_{ik}^n End-of-day inventory of product k at depot i.
- q_{ik}^n Sales of product k at depot i.
- $r_{ik,l}^n$ Rejected demand of product k at depot i, corresponding to the l-th part of the piece-wise linear penalty; $l \in \mathscr{I}$.

Then solve

$$\min \sum_{k,n} \Big(\sum_j C_{jk}^n x_{jk}^n + \sum_j H_k z_{jk}^n + \sum_{ij} c_{ij}^n w_{ijk}^n + \sum_{l=1}^{3} N_{jk,l} \sum_j r_{jk,l}^n \Big) \tag{1}$$

Subject to

$$\mathbf{1}_{\mathscr{D}_1}(j)\,x_{jk}^{n} + \mathbf{1}_{\mathscr{D}_2}(j)\,x_{jk}^{\mathrm{pa}(n)} + \sum_{i\in\mathscr{D}} w_{ijk}^{n} + z_{jk}^{\mathrm{pa}(n)} = q_{jk}^{n} + \sum_{i\in\mathscr{D}} w_{jik}^{n} + z_{jk}^{n} \quad \forall j,k,n \tag{2}$$

$$\sum_{j\in\mathscr{D}}\sum_{k}\Big(x_{jk}^{n} + \sum_{i} w_{ijk}^{n}\Big) + \sum_{j\in\mathscr{D}_2}\sum_{k} x_{jk}^{\mathrm{pa}(n)} \leq U \quad \forall n \tag{3}$$

$$q_{jk}^{n} + \sum_{i} w_{jik}^{n} \leq z_{jk}^{\mathrm{pa}(n)} \quad \forall j,k,n \tag{4}$$

$$q_{jk}^{n} + \sum_{l=1}^{3} r_{jk,l}^{n} = d_{jk}^{\mathrm{t}(n)} \quad \forall j,k,n \tag{5}$$

$$\frac{q_{jk}^{n}}{d_{jk}^{\mathrm{t}(n)}} \geq g_{jk}^{n}\frac{\sum_{m\in\mathscr{N}} p^{m}\sum_{i\in\mathscr{D}} q_{ik}^{m}}{\sum_{m\in\mathscr{N}} p^{m}\sum_{i\in\mathscr{D}} d_{ik}^{\mathrm{t}(m)}} \quad \forall j,k,n : d_{jk}^{\mathrm{t}(n)} > 0 \tag{6}$$

$$\sum_{n,j,k} P_k\, x_{jk}^{n} = B \tag{7}$$

$$z_{jk}^{n} \leq M_{jk} \quad \forall n,j,k \tag{8}$$

$$x_{jk}^{n}, w_{ijk}^{n}, q_{jk}^{n}, z_{jk}^{n} \geq 0 \quad \forall n,i,j,k \tag{9}$$

$$0 \leq r_{jk,l}^{n} \leq b_l\, d_{jk}^{\mathrm{t}(n)} \quad \forall n,j,k,\ l\in\mathscr{I} \tag{10}$$

Here the objective function (1) is the financial representation of the operational activities; the first component is the cost of transporting oil products to the different depots, the second component is holding costs at different depots (mostly capital costs); the third component shows the inter-depot transportation costs and the last term describes the cost associated with rejecting demand.

Constraints (2) maintain the flow of products in the depots. Here, $\mathbf{1}_A(x)$ denotes the *indicator function* of set A, i.e. it is equal to one if $x \in A$ and zero otherwise.

Constraints (3) enforce truck capacity. Here the first part is the truck capacity utilized for all trucks staring out that day, be that from a refinery or a depot. For depots two days away from the refineries we have a second part representing the previous day's purchases which are on the way.

Constraints (4) say that oil sent to other depots plus oil sold to customers must come from inventory. In other words, the incoming volumes x_{jk}^{n} and w_{ijk}^{n} cannot be used the same day they arrive—for example because they arrive in the evening.

Constraints (5) make sure that sales plus rejections add up to demand. Since the rejections are non-negative, it also ensures that the sales do not exceed the demand.

Constraints (6) make sure that we overall treat all depots fairly. Note that the numerator of the fraction in the right-hand side is equal to the *total expected sale* of product k, while the denominator is equal to the *total demand* for the product. Hence, the constraint says that if we satisfy on average $p\%$ of the total demand for product k, then we must satisfy at least $g_{jk}^{n}\,p\%$ of the demand in each depot j and node n.

Constraint (7) shows the limited budget available to NOC to purchase oil from India. This budget is available in foreign currencies. We use equality as we know that the budget is always too low to satisfy all demand. As this is a cost minimization model, this is our way of expressing that all sales are worthwhile. These constraints, combined with conservation of flow (2), make sure that all we buy is sold, nothing ends up permanently in inventory. In reality prices vary a bit over refineries. However, had we included that in the model, we would have ended up with a difficulty. Since the NOC budget is not big enough to cover all demand, the model would buy as much oil as possible in order to avoid penalties for lost demand. And that would mean buying from the cheapest refinery

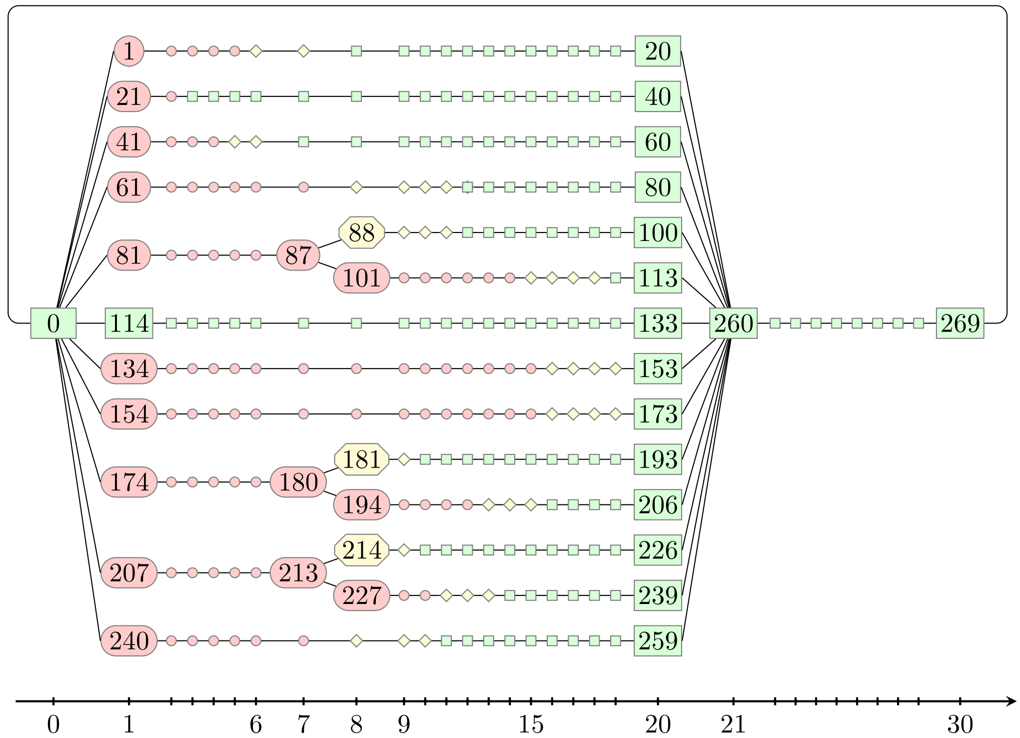

Figure 3: Scenario tree used in the numerical tests. Nodes without displayed numbers are numbered consecutively from the previous given number. Circles/ellipses represent nodes with an ongoing event, octagons and diamonds the recovery nodes and finally rectangles represent the normal nodes, i.e. nodes where the inventory is fixed to the steady-state level.

before re-distributing it all over Nepal. This would take place as long as the extra transportation and inventory costs did not outweigh the saved penalties, which they would not in the base case. To avoid this, which certainly does not describe reality, we use prices P_k which do not vary over refineries, and instead include the differences between depot prices as a part of the transportation cost C^n_{jk}. This eliminates the problem.

Finally, constraints (8) guarantee that the depots will not exceed their holding capacities, constraints (9) insure non-negative decision variables and constraints (10) limit the size of the rejections.

6 Stochastic LP formulation

The model we develop in this section is a stochastic multi-period model representing the steady-state of the operation. We use scenarios as outlined in Figure 3, which is a specific case of the more generic Figure 2. Note that because of the looping-back, it is technically not a scenario tree. We will, however, still use some of the scenario-tree terminology and call the nodes in the period before the converging nodes *leaves*, even if technically they are not.

At this point we might want to properly define our first-stage variables. Focus then on node 260: whatever is the inventory level in node 260 is also enforced during the quiet period (nodes 261 through 269, as well as in the no-disturbance scenario: nodes 0 and 114 to 133). Further, the same inventory is enforced in all leaf nodes. The idea is that after an event is over, the scenario is given some time to recover from the event, bringing inventories back to “normal” (i.e. that of node 260).

Table 1: Information about the scenario events. For each scenario, the table shows information about the start and end of the event, plus the start of the enforced normal (steady-state) state. The information is presented in terms of time and nodes—node that we do not show the time of the start of the event, as they all start in the first period ($t = 1$).

Scen.		Time		Nodes		
No.	Code	Ev. End	Nor. Start	Ev. Start	Ev. End	Nor. Start
1	R2	5	8	1	5	8
2	R4	2	3	21	22	23
3	R3	4	7	41	44	47
4	R1	7	12	61	67	72
5	R5a	7	12	81	87	92
6	R5b	14	19	81	107	112
7	N	.	0	114	.	0
8	R6	15	20	134	148	153
9	S1	15	20	154	168	173
10	P1a	7	10	174	180	183
11	P1b	12	16	174	198	202
12	P2a	7	10	207	213	216
13	P2b	10	14	207	229	233
14	P3	7	11	240	246	250

However, a problem occurs here as a result of the chosen modelling approach. Within a scenario, but after the event is over, the model "knows" there will be no disturbances. Hence, it brings inventory down to a minimal level for a while, and then builds it up in time for its leaf node. This does not reflect reality. Hence, we have added another set of nodes that must have the same inventory levels as node 260. These are found, to some extent by trial and failure, by making sure that each scenario has enough time (but not more than that) to rebuild inventory. Our first stage variables are the inventories in all nodes where inventory is forced to be equal to that of node 260.

Event durations and starts of the fixed-inventory periods for each scenario are given in Table 1, both in terms of time and node numbers. From the table we can, for example, read that the event in the third scenario starts in node 41 in period 1 and ends in node 44 in period 4. After that, the scenario is given 3 periods to recover from the event, before it is forced to have the steady-state inventory from node 47 at time 7. If we combine information from the table with Figure 3, we can see that scenarios 10 and 11 start as one, in node 174. The difference is that in scenario 10, the event ends in node 180 in period 7, while in scenario 11 it continues up to period 12. This means that the two scenarios differ from period 8 onwards, as we can see in Figure 3.

At this point, we are ready to present the stochastic model. Since most of the notation is the same as in the deterministic case, we only present the extra things needed for the stochastic case:

- τ Root node, i.e. the node just before the start of the event. This is node 0 in Figure 3.
- α Converging node, i.e. the node where all scenarios converge. This is node 260 in Figure 3.
- $\mathscr{L}$ Set of leaf nodes, i.e. the nodes just before the converging node; $\mathscr{L} \subset \mathscr{N}$. In our case, these are the nodes at $t = 20$.
- $\mathscr{C}$ Set of control nodes, i.e. the nodes where inventories must be the same as in node α; $\mathscr{C} \subset \mathscr{N}$. Note that these steady-state inventories are our first stage variables, as just outlined.
- p^n Probability of node n.
- $\mathscr{S}$ Set of scenarios, i.e. paths from the root to the final node (node 269). The probability of a scenario is given by the probability of its leaf node.
- $\mathscr{N}_s$ Set of nodes belonging to scenario $s \in \mathscr{S}$. Note that a node can belong to more than one scenario; in particular, $\{\tau, \alpha\} \subset \mathscr{N}_s$ for all $s \in \mathscr{S}$.

The objective is then to solve

$$\min \sum_n p^n \sum_k \Big(\sum_j C^n_{jk} x^n_{jk} + \sum_j H_k z^n_{jk} + \sum_{ij} c^n_{ij} w^n_{ijk} + \sum_{l=1}^{3} N_{jk,l} \sum_j r^n_{jk,l} \Big) \tag{11}$$

Subject to

$$\mathbf{1}_{\mathscr{D}_1}(j)\, x^n_{jk} + \mathbf{1}_{\mathscr{D}_2}(j)\, x^{\mathrm{pa}(n)}_{jk} + \sum_{i \in \mathscr{D}} w^n_{ijk} + z^{\mathrm{pa}(n)}_{jk} = q^n_{jk} + \sum_{i \in \mathscr{D}} w^n_{jik} + z^n_{jk} \quad \forall j,k,\ \forall n \in \mathscr{N} \setminus \{\alpha\} \tag{12}$$

$$\mathbf{1}_{\mathscr{D}_1}(j)\, x^\alpha_{jk} + \mathbf{1}_{\mathscr{D}_2}(j)\, x^n_{jk} + \sum_{i \in \mathscr{D}} w^\alpha_{ijk} + z^n_{jk} = q^\alpha_{jk} + \sum_{i \in \mathscr{D}} w^\alpha_{jik} + z^\alpha_{jk} \quad \forall j,k,\ \forall n \in \mathscr{L} \tag{12c}$$

$$\sum_{j \in \mathscr{D}} \sum_k \Big(x^n_{jk} + \sum_i w^n_{ijk} \Big) + \sum_{j \in \mathscr{D}_2} \sum_k x^{\mathrm{pa}(n)}_{jk} \le U \quad \forall n \in \mathscr{N} \setminus \{\alpha\} \tag{13}$$

$$\sum_{j \in \mathscr{D}} \sum_k \Big(x^\alpha_{jk} + \sum_i w^\alpha_{ijk} \Big) + \sum_{j \in \mathscr{D}_2} \sum_k x^n_{jk} \le U \quad \forall n \in \mathscr{L} \tag{13c}$$

$$q^n_{jk} + \sum_i w^n_{jik} \le z^{\mathrm{pa}(n)}_{jk} \quad \forall j,k,\ \forall n \in \mathscr{N} \setminus \{\alpha\} \tag{14}$$

$$q^\alpha_{jk} + \sum_i w^\alpha_{jik} \le z^n_{jk} \quad \forall j,k,\ \forall n \in \mathscr{L} \tag{14c}$$

$$q^n_{jk} + \sum_{l=1}^{3} r^n_{jk,l} = d^{\mathrm{t}(n)}_{jk} \quad \forall n,j,k \tag{15}$$

$$\frac{q^n_{jk}}{d^{\mathrm{t}(n)}_{jk}} \ge g^n_{jk} \frac{\sum_{m \in \mathscr{N}} p^m \sum_{i \in \mathscr{D}} q^m_{ik}}{\sum_{m \in \mathscr{N}} p^m \sum_{i \in \mathscr{D}} d^{\mathrm{t}(m)}_{ik}} \quad \forall j,k,\ \forall n : d^{\mathrm{t}(n)}_{jk} > 0 \tag{16}$$

$$\sum_{j,k} P_k \sum_{n \in \mathscr{N}_s} x^n_{jk} = B \quad \forall s \in \mathscr{S} \tag{17}$$

$$z^n_{jk} \le M_{jk} \quad \forall n,j,k \tag{18}$$

$$z^n_{jk} = z^\alpha_{jk} \quad \forall j,k,\ \forall n \in \mathscr{C} \tag{19}$$

$$x^\tau_{jk} = \frac{\sum_{n \in \mathscr{C}} p^n x^n_{jk}}{\sum_{n \in \mathscr{C}} p^n} \quad \forall j \in \mathscr{D}_2,\ \forall k \tag{20}$$

$$x^n_{jk}, w^n_{ijk}, q^n_{jk}, z^n_{jk} \ge 0 \quad \forall n,i,j,k \tag{21}$$

$$0 \le r^n_{jk,l} \le b_l d^{t(n)}_{jk} \quad \forall n, j, k, \; l \in \mathscr{I} \tag{22}$$

Here the objective function (11) is the financial representation of the operational activities; it is the same as in the deterministic model except for the obvious addition of probabilities. Constraints (15), (16), (18), (21) and (22) are exactly the same as their deterministic counterparts (5), (6), (8), (9) and (10).

For constraints that point one period back, we have to treat the converging node separately, since it does not have one given parent, but a whole set of parent nodes—the set of leaves. In the model, these pairs of constraints share the same equation number, with the converging-node variant having 'c' added to the number. Apart from this, these constraints remain unchanged from the deterministic case. In particular, constraints (12)–(14) are the same as respectively (2)–(4).

The budget constraint (17) differs from its deterministic counterpart (7) in the sense that we require it to hold for every scenario.

The first constraints without any deterministic counterpart are constraints (19), which take care of our first stage variables by forcing inventory in the *control nodes* to be equal to the steady-state inventory. The set of control nodes includes all the nodes in the quiet periods, the converging node, the leaf nodes, and those nodes where inventory would otherwise dip to minimal levels as explained earlier.

Finally, constraints (20) make sure that purchases in the root node for depots which are two days away from their refineries equal the average of the purchase in the steady-state nodes of the model. This is included to make sure that extra-ordinary purchases are not made on the day preceding the event. Such purchases would contradict the logic of the model.

6.1 Random events

In this section, we discuss the different possible random events that may occur and then present the scenarios used in our numerical tests. Note that the scenario names are the same as in Table 1.

Among many possible uncertainties in the supply chain of NOC, we discussed a few which occur with a reasonable frequency. Landslides often block road links between depots. Another common event is strikes called by truck owners or drivers. This can be isolated to a region or affect the whole system. Accidents may also block parts of the network. Political disturbances have been a major reason for blocked road in recent years. Again this can be local or affect the whole network. Breakdown of refineries is also a source of uncertainties that NOC faces. We consider a few scenarios of these random events for our analysis.

Scenarios due to road problems.

Scenario R1. The road link between Kathmandu and Amlekhganj breaks down due to landslide for a week. Here the inter-depot transport to Kathmandu is affected from Amlekhganj and Biratnagar. These are now routed through Birgunj. Also because of this breakdown the supply to Kathmandu from Raxaul is via Birgunj.

Scenario R2. The road to Kathmandu closes down due to an accident for 5 day and makes Kathmandu isolated from the rest of the country. In this scenario supply to and all inter-depot movement from and to Kathmandu are stopped.

Scenario R3. The road link between Kathmandu and Amlekhganj breaks down due to strike of truck drivers for 3 days. In this scenario the inter-depot link from Amlekhganj and Biratnagar is affected and now will be routed through Birgunj. Also the supply to Kathmandu from Raxaul will be through Birgunj.

Scenario R4. The road link between Raxaul refinery and Amlekhganj depot breaks down due to accident for 2 days. Here inter-depot movement is not affected and only the supply to Amlekhganj is routed through Birgunj. In doing so the supply to reach Amlekhganj takes two days instead of one day. To be able to model this, we have to let the sets $\mathscr{D}_1, \mathscr{D}_2$ depend on node n and change the appropriate part of constraints (12) and (12c) to

$$\mathbf{1}_{\mathscr{D}_1^n}(j)\, x_{jk}^n + \mathbf{1}_{\mathscr{D}_2^{\mathrm{pa}(n)}}(j)\, x_{jk}^{\mathrm{pa}(n)}\,.$$

Note the $\mathrm{pa}(n)$ index on the $\mathscr{D}_2$ set. This way, we can properly model the fact that there is no delivery on the first day and two deliveries on the third day: the delayed delivery from the second day of the break-down, and the one-day delivery ordered on the third day.

Scenario R5. The road link between Biratnagar depot and Barauni refinery breaks down due to flood for one or two weeks with equal probability. Since the Barauni refinery supplies all products except ATF to Biratnagar, the new source for Biratnagar for those products will be Raxaul, as this is the nearest refinery available.

Scenario R6. The link between Allahabad refinery and Bhairahawa depot breaks down due to damage of a bridge for 15 days. Since Allahabad is the source for ATF (air fuel), the breakdown affects the flow of ATF to Bhairahawa. Here the nearest refinery or depot of IOC after Allahabad will be Raxaul and the route will be via Birgunj.

Scenario due to refinery breakdown.

Scenario S1. Barauni refinery is down for 15 days. As Barauni is down, the next nearest source point for Biratnagar is Raxaul.

Scenarios due to political disturbance.

Scenario P1. The link to the Kathmandu depot from Amlekhganj is down for one week with a probability of 60 percent or down for 12 days with probability of 40 percent.

Scenario P2. The link to the Kathmandu depot from Amlekhganj is down for one week or for 10 days with probability of 60 and 40 percent, respectively.

Scenarios P1 and P2 are structurally the same (only durations vary). This reflects the importance of this link in connection with political disturbances.

Scenario P3. All the links to Biratnagar, Kathmandu and Pokhara depots are down for one week. In this scenario these depots are not reachable from any other depots or refineries.

Here we can see that when the scenarios R1, R2, R3, R4, R6, S1, and P3 occur we know their durations and hence these scenarios can be represented in a fan structure. But scenarios R5, P1 and P2 have random durations, and hence can be represented in a tree structure. In total, we end up with the scenario structure presented in Figure 3.

7 Computational Results

The model has been implemented using AMPL modelling language , and solved using CPLEX. 9.0.0 on a 3 GHz PC with 1 GB of RAM. Solution times were mostly less than 2 minutes (the base case takes 10 seconds), with one case using 15 minutes, all with cold starts, while warm starts would mostly take just a couple of seconds.

The cases we are about to present are realistic. However, our goal is not to provide specific advice in a specific case, but rather to illustrate how optimal inventories depend on different model parameters. We have used real data, collected on the ground in Nepal, for transportation costs, inventory costs (mostly capital costs), truck capacities, network structure, and demand. Our base case for the budget covers about 78% of demand (which is reasonable), and we have used all fairness factor g^n_{jk} the same and equal to 0.7.

For penalties, we have used three intervals with $b = \{0.12, 0.20, 0.68\}$. In other words, the break points are at 12% and 32% of unsatisfied demand, which corresponds to the average level of unsatisfied demand (100% - 78% = 22%) plus/minus 10%. Base case penalties are such that penalties do not drive the solution, but enough to make sure non-deliveries are evenly distributed (the non-deliveries which are not governed by the fairness). This is the hardest parameter to set. We observe, though, that within reasonable (and large) intervals, the sensitivity to penalties is low for the other parameters at their base values.

Let us start by showing how total inventory develops over time (averaged over the events) for our base case. It is shown in Figure 4. We have distinguished between inventory (to be named *technical inventory*) that would be there even if there were no events at all (caused by (4)), and what is added due to the events (hereafter called *event inventory*), i.e. the inventory of interest for this paper. As can be seen, inventory is kept in expectation of events, then when events start, inventory drops, and is then rebuilt. Although not shown in the figure, for each inventory, there is at least one event (scenario) for which the event inventory goes to zero. If that had not been the case, we would have been keeping inventory (at a cost) that was never needed, and that could not be optimal.

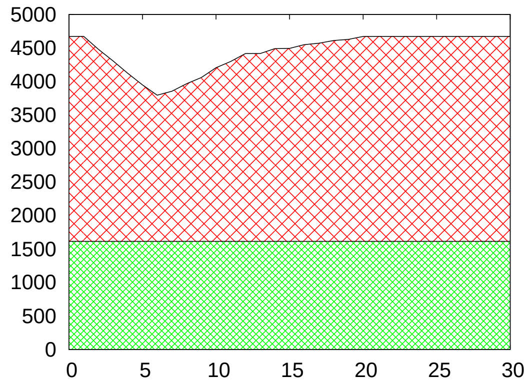

Figure 4: Optimal inventory over time for the base case. The bottom part shows technical inventory caused by (4) in the deterministic case, the top part event inventory that we keep for the random events.

7.1 Budget

Inventory is kept in anticipation of events and deliveries. If the budget is increased, we can deliver more, and this will of course increase technical inventory as deliveries have to come from inbound inventory. But also the event inventory will increase. This is simply because the amounts to be handled during events have also increased. There are two driving factors here, the penalties and the fairness. If deliveries in some important depot drops substantially during an event, the piece-wise linearity of the penalties will force inventory to prevent that from happening. But g also have some interesting effects. Some of these effects can be found in Figure 5. In the left part $g = 0.25$, in the right $g = 0.95$.

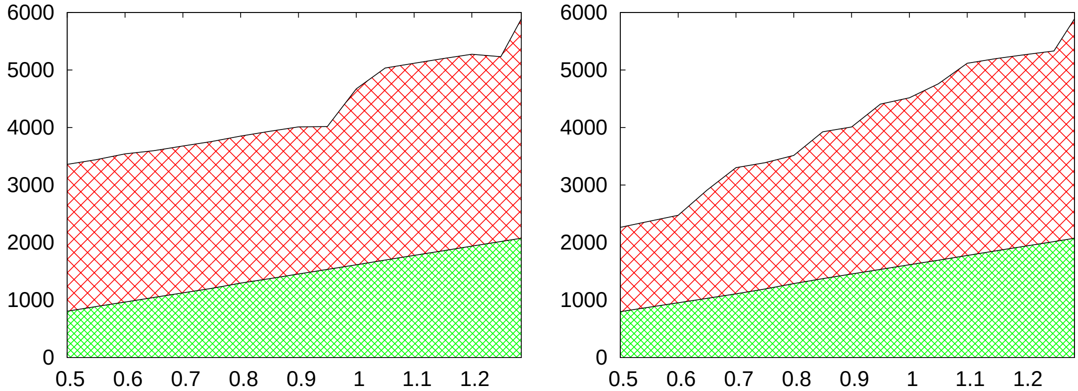

Figure 5: Steady-state inventory as function of budget for low and high g. Budget is measured relative to our base case (corresponding to 1 on the horizontal axis). The bottom part shows technical inventory caused by (4) in the deterministic case, the top part event inventory that we keep for the random events.

In the figure, we can see that for all but rather high budgets a low g yields higher inventories than a high g. How can that happen? With a low g, deliveries are driven mainly by costs and penalties, so some depots can be treated very badly. The effect is that we supply mainly Kathmandu, as it has large demands and the highest penalties (except for the first interval, where the penalties $N_{jk,l}$ are the same for a given product k over all depots j).

This reflects that Kathmandu has more critical infrastructure than the rest of the country. As g increases, we are forced to supply also other depots, so the sales in Kathmandu go down. Now, because of its location and its status as the capital of Nepal, Kathmandu is exposed both to the physical and the political disturbances—most of the scenarios harm Kathmandu and/or its neighbours. As a result, Kathmandu needs the largest event inventory relative to sales. Hence, as g forces more and more sales out of Kathmandu, the overall inventory goes down.

These results are stable over penalty levels. The reason is that in all runs Kathmandu has higher penalties than the other depots, whether the penalties are high or low, so the qualitative arguments above hold. The objective function is of course dependent on the level of the penalties, but the solutions are not. This was the intended effect of penalties.

The effects of g are reasonable. If fairness is given low importance, the pressures from the major depots, caused by their importance, will guide the deliveries. Fairness is therefore needed to reflect that NOC is not mainly there to make a profit, but to efficiently distribute oil in a socially acceptable way.

7.2 Political unrest

The relationship between the level of political unrest and steady-state inventory illustrates how the model prepares for the random events. This is illustrated in Figure 6.

We ran the model by increasing the probabilities for the political disturbances proportionally and reducing equally that of the non-event scenario, keeping the probability of all other types of risks fixed. We expected inventory levels to be monotonously increasing with the level of political unrest. This is also what we observe, but the effect is very moderate. Why is the curve so flat for all but the lowest probabilities? The reason is that most of the event inventory is driven by fairness (which acts as a constraint) and penalties. The fairness aspect does not depend on probabilities: the nodes are treated equally irrespective of their probabilities (except when zero). Penalties are of course more serious when probabilities are high, but the effect is marginal as the relative sizes of penalties are not changed, only the level. The higher is g, the more this effect is true: as soon as events exist and must be protected against, we act even if probabilities are low.

We are careful about using precise numbers here, as we are generally looking for qualitative understanding. But even so, it is worth noting that already at 0.25% probability of political unrest, the event inventory increases by 58% (from 1477 to 2336), compared to the case without any political risk. And for 16% risk, the event inventory is up 107% (from 1477 to 3060).

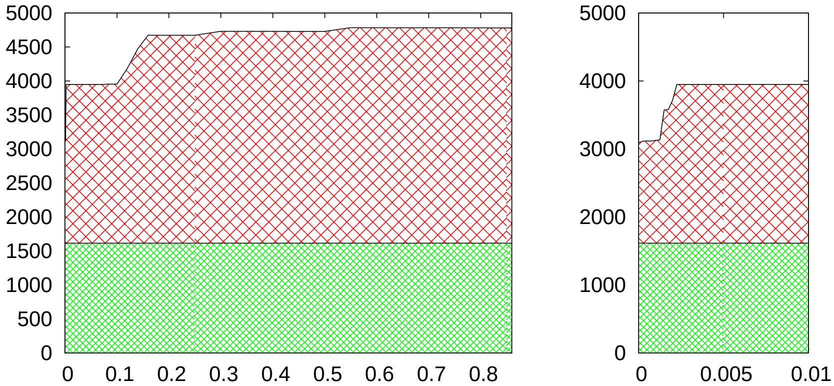

Figure 6: Steady-state inventory as a function of the probability of political unrest. The bottom part shows technical inventory caused by (4) in the deterministic case, the top part event inventory that we keep for the random events. The smaller figure to the right presents a detail view of probabilities close to zero. We can see that already at 0.25% probability, the inventory reaches a level that is sufficient for probabilities up to 10%.

Conclusion

We have formulated, solved, and analyzed the problem of distribution and inventory management in an infinite horizon problem with random disturbances in the flow network. A unique modelling approach is used to find the optimal inventory positions. It is based on looping the network back on itself and changing the time line so as to create a three stage model out of what is, in reality, an infinite horizon problem.

The present approach is unique in the sense that it can incorporate any type and intensity of disturbances in the flow network and can show their affect on the steady-state inventory positions.

Our numerical test results from Nepal show that management can get useful insights into the steady-state inventories to be maintained during normal periods in anticipation of random events. Also this approach may be used for analyzing the effects of any new plans, like adding a pipeline or a new road link, to the existing flow network.

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